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"Models of Growth with Imported Inputs"

by

Pranab Bardhan and Sydney Lewis

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MODELS OF GROWTH WITH IMPORTED INPUTS*

Pranab Bardhan
and
Sydney Lewis

I

A major part of world trade is in intermediate products and capital goods, while trade theory is mostly in terms of finished consumer goods. This is now generally recognized; Bhagwati in his survey article [3] points to this as *the* central limitation of trade theory. Imports of intermediate goods (like steel, cement, fertilizers) and machinery of different types are particularly important for most developing economies. In fact an increasing number of developing nations' imports tend to consist almost exclusively of these commodities. In this paper we construct two classes of very simple growth models, one in which imports consist solely of intermediate products, and the other in which they consist only of machinery that is different from those produced at home.

The literature on models of growth and trade is still in its infancy. In the last few years some fairly rigorous models in this area have been developed, mostly belonging to the class represented by papers of Oniki and Uzawa [7] and Bardhan [1] [2]. These authors extend the familiar two-sector two-factor closed-economy model of recent literature on neo-classical growth theory to the case where there is balanced trade in the two goods between two countries. They analyze the changing patterns of specialization as capital and labour accumulate in the international economy and also how from any given initial conditions the capital-labour ratios in the two economies and the terms of trade approach their

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[*] We have benefited from discussion with Miguel Sidrauski. All errors are, of course, exclusively ours.

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steady-state levels.

While neo-classical general-equilibrium growth models are gradually being extended to include international trade, there is another branch of international economics, particularly in discussions of economic development and international aid, where a somewhat different kind of model, formally less sophisticated but no less interesting, is increasingly gaining prominence, incorporating some of the 'stylized' facts of trade and development and oriented more to a programming formulation. This literature might be represented by papers of Chenery and Bruno [4], Chenery and Strout [5], McKinnon [6] and others. One of the 'stylized' features embedded in this literature is a sluggish world demand for exports (particularly of primary commodities) acting as a bottleneck on growth for a developing economy. In this paper we develop models in which the home economy faces a world demand for its exports that is price-inelastic and growing at an exogenously determined rate (which may be lower than the domestic rate of growth of population, thereby depressing the long-run growth rate of this economy.[1]) We analyze the time path of output, capital stock, imports and terms of trade in these models and also find out the conditions for their convergence to the steady-state levels. By introducing the export demand bottleneck we thus try to contribute towards building a bridge between these two kinds of models of trade and growth in the literature. By considering in Section III imports of foreign machinery which is different from domestically produced capital goods, we also try to take account of one of the important factors in the development literature and at the same time go beyond the Oniki-Uzawa-Bardhan models where the problem of allocating savings among heterogenous capital goods is ignored.

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[1] In the usual neo-classical growth model the long-run growth rate of the economy is equal to the "natural" rate of growth unaffected by demand conditions. But in our model this is influenced by conditions of world demand for exports, because the latter affect the rate of growth of availability of imported inputs used in domestic production.

II

Since our models involve these extra complications, for getting manageable qualitative conclusions we have simplified them somewhat in other respects. For example, throughout this paper we assume Cobb-Douglas production functions as well as a constant elasticity of export demand with respect to price. Without these assumptions the analysis would certainly get more messy, but our basic framework need not be discarded. (We might mention here that Cobb-Douglas production functions, apart from their linear-logarithmic nature facilitating all calculations, possess one extra desirable property in the context of our present problem, viz. it makes domestic factors *complementary with* imported inputs of intermediate products or capital goods in production[2]: this is certainly in line with some of the ideas in the development literature). Then again we assume throughout that domestic output (that is consumable, exportable and accumulatable) is produced on the same production function and that the home economy produces no substitute for the imported inputs. These are obviously simplifying assumptions, a more general model should add to the number of production sectors and analyze all possible patterns of specialization--in carrying out this generalization one should be helped by the analytic methods developed in Oniki and Uzawa [7] and Bardhan [1] [2].

In this Section we take intermediate products as the only imported input. The production function for domestic output Q is given by

$$Q = K^{\alpha}V^{\beta}L^{1-\alpha-\beta} \tag{1}$$

where K is capital, V is the amount of imported intermediate product, L is

---

[2] In the sense that increased use of imported inputs raises the marginal product of domestic factors of production.

labour and the exponents are given positive constants. Output per unit of labour, q, is thus given by

$$q = k^{\alpha}v^{\beta} \tag{2}$$

where k and v are K and V in terms of labour. National income per unit of labour, y, is given by

$$y = Pq - v \tag{3}$$

where P is the price of domestic output and the imported good taken as the numéraire. Under free trade P also stands for the terms of trade in this economy. The price of the imported input being equal to the value of its marginal product, we have

$$1 = P\cdot\frac{\partial Q}{\partial V} = P\cdot\beta k^{\alpha}v^{\beta-1}. \tag{4}$$

The balance of trade equation is given by

$$PX = V \tag{5}$$

where X is the amount of domestic output exported. We assume that the world demand for this export of our home economy is price-inelastic and growing at an exogenously given rate λ, so that

$$X = P^{\eta}e^{\lambda t} \quad , \text{ with } \eta<0,\ 1+\eta>0. \tag{6}$$

where η is the constant price-elasticity of demand.

We shall assume that labour grows at an exponential rate μ, so that

$$L(t) = L(0)e^{\mu t}. \tag{7}$$

Without any loss of generality, let us put L(0) = 1. Now from (4), (5) and (6),

$$v = P^{1+\eta}e^{(\lambda-\mu)t}. \tag{8}$$

We have the simplest savings function whereby a constant fraction s of national income is saved and invested, so that

$$\frac{\dot{K}}{K} = \frac{sy}{Pk} \, . \tag{9}$$

We are ignoring depreciation of capital, but proportional depreciation may be easily introduced. Using (2), (3), and (4), (9) may be rewritten as

$$\frac{\dot{K}}{K} = s \cdot k^{\alpha-1} v^{\beta} (1-\beta) . \tag{10}$$

Since in the steady state capital should grow at a constant rate, (10) immediately suggests that in the steady state $\underline{k} \equiv \frac{k^{1-\alpha}}{v^{\beta}}$ is a constant.

From (4) and (8),

$$\frac{\dot{v}}{v} = \frac{\lambda - \mu - \alpha(1+\eta)\frac{\dot{k}}{k}}{1 - (1+\eta)(1-\beta)} \tag{11}$$

where the denominator is positive, since 1 > 1+η > 0. We are now in a position to write down the basic differential equation of the model:

$$\frac{\dot{\underline{k}}}{\underline{k}} = (1-\alpha)\frac{\dot{k}}{k} - \beta\frac{\dot{v}}{v} = \frac{\dot{k}}{k}\left[(1-\alpha) + \frac{\alpha\beta(1+\eta)}{1-(1-\beta)(1+\eta)}\right] + \frac{(\mu-\lambda)\beta}{1-(1+\eta)(1-\beta)}$$

$$= \frac{\dot{K}}{K}\frac{[\beta-\eta(1-\alpha-\beta)]}{[1-(1-\beta)(1+\eta)]} - \frac{[\lambda\beta-\mu\eta(1-\alpha-\beta)]}{[1-(1-\beta)(1+\eta)]} . \tag{12}$$

Using (10) in (12),

$$\frac{\dot{\underline{k}}}{\underline{k}} = \frac{s(1-\beta)}{\underline{k}} \; \frac{[\beta(1+\eta) - \eta(1-\alpha)]}{[1-(1-\beta)(1+\eta)]} - \frac{[\lambda\beta-\mu\eta(1-\alpha-\beta)]}{[1-(1-\beta)(1+\eta)]} . \tag{13}$$

From (12) and (13) we can characterize the time-path of this economy in terms of Figure 1.

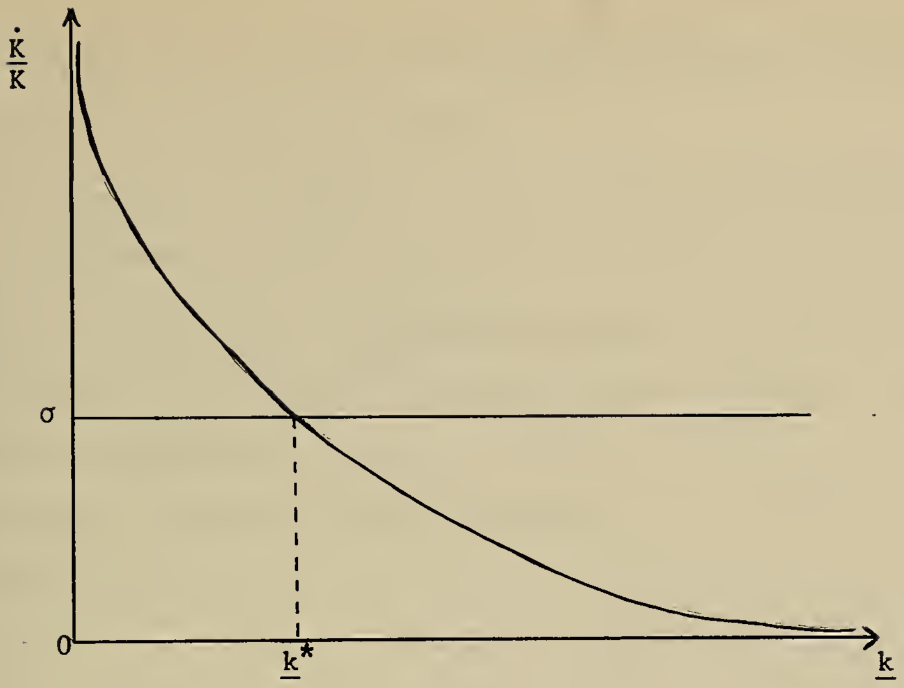

Figure 1

In Figure 1, the long-run equilibrium rate of growth of capital is $\sigma = \frac{\lambda\beta-\mu\eta(1-\alpha-\beta)}{\beta-\eta(1-\alpha-\beta)} > 0$, and $\underline{k}^*$ is the steady-state level of $\underline{k}$. Since $\frac{\dot{K}}{K} = \frac{s(1-\beta)}{\underline{k}}$ is a uniformly declining function of $\underline{k}$ and asymptotic to the two axes, the steady state exists, is unique and globally stable (i.e. from any given initial $\underline{k}$ the economy tends to converge to $\underline{k}^*$).

It is interesting to note that if the exogenous rate of growth in world demand for the home country's exports is smaller than its rate of growth of population (which is observed in a number of developing countries today, India being a major example), the long-run equilibrium rate of growth of capital is *lower* than its 'natural' rate of growth (in this case, the rate of growth of population). In other words, if $\mu > \lambda$, $\sigma < \mu$.[3] This in some sense highlights

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[3] One might note that $\sigma = \frac{\lambda\beta}{\beta-\eta(1-\alpha-\beta)} - \frac{\mu\eta(1-\alpha-\beta)}{\beta-\eta(1-\alpha-\beta)}$ is a weighted average of λ and μ.

It should be pointed out that σ is the long-run equilibrium rate of growth of capital and not that of national income. The latter, $\frac{\dot{Y}}{Y} = \frac{\dot{P}}{P} + \frac{\dot{K}}{K}$ in the steady state. Since from (14) $\frac{\dot{P}}{P} < 0$ in the steady state for $\mu > \lambda$, $\frac{\dot{Y}}{Y} < \sigma$. In other words, the long-run equilibrium rate of

(continued on page 7)

the depressing impact of a sluggish world demand for exports on the long-run equilibrium rate of growth of an economy.

One might also note that $\frac{d\sigma}{d\lambda} > 0$, i.e. the smaller the exogenous rate of growth of world demand, the smaller the long-run growth rate, and that for $\lambda < \mu$, $\frac{d\sigma}{d\eta} < 0$, i.e. the smaller the absolute value of the price-elasticity of demand for exports, the smaller is the long-run rate of growth. Both of these results are quite plausible.

What happens to terms of trade over time?

From (4) and (11),

$$\frac{\dot{P}}{P} = \alpha\mu - (\mu-\lambda)(1-\beta) - \alpha\frac{\dot{K}}{K}. \tag{14}$$

For $\underline{k} \lesseqqgtr \underline{k}^*$, $\frac{\dot{K}}{K} \gtreqqless \sigma$, and it is easy to see from (14) that $\frac{\dot{P}}{P} < 0$ if $\mu > \lambda$; so terms of trade for the home country decline over time for $\underline{k}$ not exceeding its steady-state level. Imports as a proportion of national income, however, remain constant over time, since $\frac{V}{Y} = \frac{1}{\frac{PQ}{V} - 1} = \frac{\beta}{1-\beta}$.

The so-called 'golden rule' for maximum steady-state level of per capita consumption in this model appears somewhat different from the usual condition of marginal product of capital equalized to the rate of growth, but this has an easy interpretation. Per capita consumption in this economy is given by

$$c = \frac{y}{P} - \frac{\dot{K}}{L}. \tag{15}$$

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3 (continued) growth of national income is less than that of capital, and à fortiori less than the rate of growth of population. So in this model in the steady-state per capita income is declining. Certainly this kind of a situation cannot go on forever in the real world; besides, there are offsetting factors like technical progress, foreign aid and so on. The major purpose of this paper is only to focus on the demand limitation on growth.

It may also be easily checked that the equilibrium rate of growth of national income is smaller, the smaller is the exogenous rate of growth of world demand and/or the smaller is the absolute value of price-elasticity of demand for exports.

Since in the steady state $\frac{\dot{K}}{K} = \sigma$, (15) may be rewritten as

$$c = \frac{y}{P} - \sigma k. \tag{16}$$

The 'golden rule' is therefore given by the following condition:

$$\frac{d(\frac{y}{P})}{dk} = \sigma. \tag{17}$$

Since $\frac{y}{P}$ is per capita real income in this model, (17) means that the 'golden rule' is to equalize the marginal contribution of capital (direct as well as indirect through its impact on amount of imported input used) to real income with the rate of growth of capital. Since from (2) and (3), $\frac{y}{P} = (1-\beta)k^{\alpha}v^{\beta}$, (17) implies that

$$(1-\beta)k^{\alpha-1}v^{\beta}[\alpha + \beta \frac{dv}{dk} \frac{k}{v}] = \sigma. \tag{18}$$

From (4) and (8), for a given t

$$\frac{dv}{dk} \frac{k}{v} = \frac{-\alpha(1+\eta)}{1-(1+\eta)(1-\beta)} . \tag{19}$$

Using (19), (18) may be rewritten as

$$\frac{\alpha k^{\alpha-1}v^{\beta}(1-\beta)(-\eta)}{1-(1+\eta)(1-\beta)} = \sigma \tag{20}$$

From (20) it is easy to see that the 'golden rule' in this model implies that the rate of growth of capital, σ, should be smaller than $k^{\alpha-1}v^{\beta}$, which is the direct marginal product of capital in the production function.

Before ending this Section we might note that in (6) we have assumed that the world demand for the export of our home economy is price-inelastic (i.e. $1 + \eta > 0$), but nowhere in our calculations in this Section we have used this. Although this assumption is going to play an important role in our next Section, all the qualitative conclusions made so far are valid even if export demand is price-elastic (or $1 + \eta \leqq 0$). Those conclusions are, however, sensitive to the

assumption of world demand growing at an exogenous rate lower than the rate of growth of population. We might briefly indicate here how these conclusions will change if we relax this assumption (after all, there are some underdeveloped countries--at least the oil-exporting ones--for which λ may be well above μ). The proof of existence, uniqueness and global stability of long-run equilibrium is, of course, unaltered. But the equilibrium rate of growth of capital, σ, is now larger than (equal to) μ, since λ is larger than (equal to) μ. Besides, $\frac{d\sigma}{d\lambda}$ is still positive, but $\frac{d\sigma}{d\eta} \gtreqq 0$ for $\lambda \gtreqq \mu$. In the steady state terms of trade improve (remain constant) for λ larger than (equal to) μ. The steady-state per capita income is also increasing for $\lambda > \mu$. The 'golden-rule' discussion remains unaltered.

III

In this Section we analyze another model which is similar to the preceding one in many respects, but now imports consist only of foreign machines which are different from domestically produced machines.

The production function for domestic output Q is given by

$$Q = K_1^{\alpha} K_2^{\beta} L^{1-\alpha-\beta} \tag{21}$$

where K_1 is domestic capital and K_2 is foreign capital all of which is imported. Output per capita, q, is thus

$$q = k_1^{\alpha} k_2^{\beta}. \tag{22}$$

The rental rates, $R_i (i=1,2)$, on capital are given by the marginal value productivity conditions, or

$$R_1 = P \cdot \frac{\partial q}{\partial k_1} = P \cdot \alpha k_1^{\alpha-1} k_2^{\beta} \tag{23}$$

$$R_2 = P \cdot \frac{\partial q}{\partial k_2} = P \cdot \beta k_1^{\alpha} k_2^{\beta-1} \tag{24}$$

where P, as before, is the price of domestic output (under free trade, also the terms of trade) with the imported good taken as numéraire. The balance of trade equation is given by

$$PX = \dot{K}_2 . \tag{25}$$

Physical depreciation of both types of capital will be ignored. As before, labour grows at an exponential rate μ, with L(0) = 1, and a constant proportion, s, of national income is saved and invested so that

$$sPQ = P\dot{K}_1 + \dot{K}_2. \tag{26}$$

We have yet to introduce an equation that determines the demand for either type of capital (i.e. allocation of investment between the two capital goods). Suppose demand for capital stock of either type depends only on the value of total wealth and the two own rates of return on capital. More specifically, we assume the following demand function for K_2:[4]

$$K_2 = W \cdot g(\frac{R_1}{P}, R_2) \tag{27}$$

where $W = PK_1 + K_2$ is the total value of wealth, the wealth elasticity of demand

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[4] In this model if (27) is the market-clearing equation for K_2, the market for K_1 will be automatically cleared so that we do not have to introduce a separate demand function for K_1.

The justification of this stock demand function (27) is based on considerations of portfolio diversification discussed by Tobin [9]. A similar demand function, with money and capital as assets, is used in a growth model by Tobin [8], p. 679.

We have checked that if instead of this stock demand function we had taken a flow demand function like

$$\frac{\dot{K}_2}{sPQ} = g(\frac{R_1}{P}, R^2_2)$$

where the two own rates of return determine the proportion of savings currently allocated to K_2, with $\dot{K}_1 \geqq 0$ and $\dot{K}_2 \geqq 0$, all the stability conditions and other properties of this model remain similar. See also f.n. 5.

for K_2 is assumed unity (for simplification). It is plausible to suppose that demand for K_2 varies positively with changes in its own rate of return, R_2, so that $g_2 = \frac{\partial g}{\partial R_2} > 0$ and that it varies inversely with changes in the own rate of return on the other capital, so that $g_1 = \frac{\partial g}{\partial (\frac{R_1}{P})} < 0$.

As in the preceding Section, we assume that the world demand for exports of the home economy is price-inelastic and growing at an exogenously given rate. Let us for the time being assume that this given rate of growth of demand happens to be equal to the home country's rate of growth of population (i.e. $\mu = \lambda$). We shall refer to the more interesting cases of $\mu \neq \lambda$ at the end of this Section. Thus we have

$$X = P^{\eta} e^{\mu t} \quad , \quad \text{with} \quad \eta < 0 \text{ and } 1 + \eta > 0. \tag{28}$$

Using (23) and (24), let us rewrite (27) in per capita terms as

$$[Pk_1 + k_2] g(\alpha k_1^{\alpha-1} k_2^{\beta}, \; P\beta k_1^{\alpha} k_2^{\beta-1}) - k_2 = 0. \tag{29}$$

This gives P, the terms of trade, as a function of k_1 and k_2. From (29), the elasticity of P with respect to k_1, or

$$\frac{\partial P}{\partial k_1} \frac{k_1}{P} = E_{p1} = - \frac{[(1-g) + e_1(1-\alpha) + e_2\alpha]}{(1-g + e_2)} < 0 \tag{30}$$

where $e_2 = \frac{g_2 R_2}{g}$ is the elasticity of the g function (the function that gives the proportion of total wealth kept in terms of imported capital, K_2) with respect to its own rate of return, R_2; and $e_1 = - \frac{g_1 R_1}{Pg}$ is the (positively defined) elasticity of the g function with respect to the own rate of return on the other type of capital, K_1. Since, from (29), $1 > g > 0$ (with positive amounts of both capital), E_{p1} is negative. This means that terms of trade for the home country decline with accumulation per capita of domestic capital.

From (29), again, the elasticity of P with respect to k_2, or

$$\frac{\partial P}{\partial k_2}\frac{k_2}{P} = E_{p2} = \frac{[(1-g) + e_1\beta + e_2(1-\beta)]}{(1-g) + e_2} > 0. \tag{31}$$

This means that terms of trade improve with accumulation per capita of foreign capital. Since $E_{p1} + E_{p2} \gtreqless 0$ for $e_2 - e_1 \gtreqless 0$, the terms of trade will improve under this condition if k_2 grows at a higher rate than k_1.

We may now write down the two differential equations in the system. From (25), (28) and (29) and letting $L(0) = 1$, the rate of accumulation of k_2 is given by (32).

$$\frac{\dot{k}_2}{k_2} = \frac{P^{1+\eta}}{k_2} - \mu = \Phi(k_1,k_2). \tag{32}$$

From (22), (26) and (32), the rate of accumulation of k_1 is given by (33).

$$\frac{\dot{k}_1}{k_1} = sk_1^{\alpha-1}k_2^{\beta} - \frac{P^{\eta}}{k_1} - \mu = \Psi(k_1,k_2). \tag{33}$$

Let us now analyze the properties of these two differential equations. Since $E_{p1} < 0$ from (30) and $1 + \eta > 0$, in (32)

$$\Phi_1 = \frac{\partial\Phi}{\partial k_1} < 0. \tag{34}$$

Since from (31), $E_{p2} \leqq 1$ for $e_2 \geqq e_1$, $(1+\eta)E_{p2} < 1$ and, therefore, in (32)

$$\Phi_2 = \frac{\partial\Phi}{\partial k_2} < 0. \tag{35}$$

Thus under the sufficient condition $e_2 \geqq e_1$, i.e. the own-elasticity in the g function is larger than or equal to the cross-elasticity,

$$\left(\frac{dk_1}{dk_2}\right)_{\Phi=0} = -\frac{\Phi_2}{\Phi_1} < 0 \tag{36}$$

so that the $\dot{k}_2 = 0$ curve is downward-sloping in the (k_1,k_2) plane, as shown in Figure 2.

From (33),

$$\Psi_1 = \frac{\partial\Psi}{\partial k_1} = \frac{1}{k_1^2}\,[-(1-\alpha)sk_1^{\alpha}k_2^{\beta} + P^{\eta}\{1-\eta E_{p1}\}\,] \qquad (37)$$

Since from (33) it follows that along $\Psi = 0$, $sk_1^{\alpha}k_2^{\beta} > P^{\eta}$,

$$\left(\Psi_1\right)_{\Psi=0} < 0 \qquad (38)$$

if $\eta E_{p1} \geqq \alpha$; from (30) it is easy to see that a (strongly) sufficient condition for the latter is that the absolute value of the price elasticity of demand for exports is large enough, i.e.

$$|\eta| \geqq \frac{\alpha e_2}{\alpha e_2 + (1-\alpha)e_1}\,. \qquad (39)$$

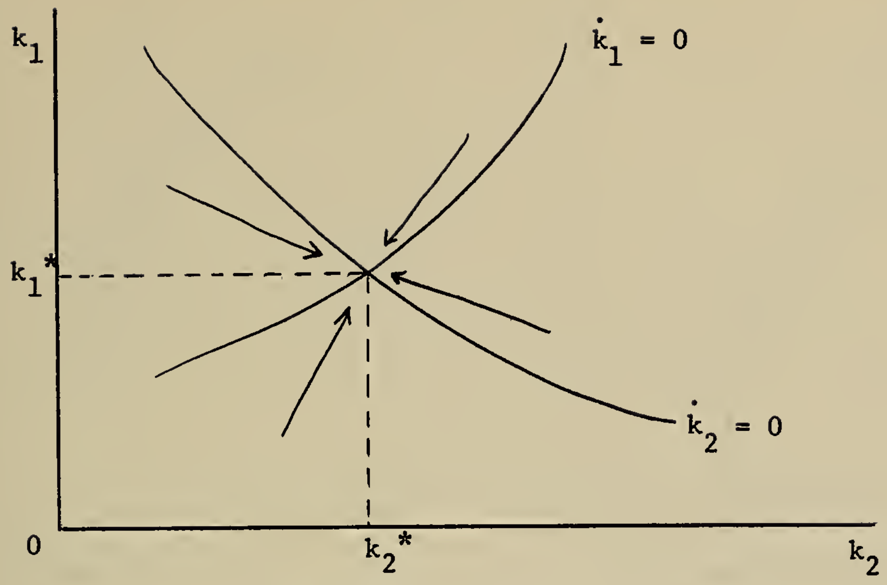
Since from (31) $E_{p2} > 0$, it immediately follows from (33) that

$$\Psi_2 = \frac{\partial\Psi}{\partial k_2} > 0. \qquad (40)$$

Thus under the sufficient condition of (39),

$$\left(\frac{dk_1}{dk_2}\right)_{\Psi=0} = -\left(\frac{\Psi 2}{\Psi_1}\right)_{\Psi=0} > 0 \qquad (41)$$

so that the $\dot{k}_1 = 0$ curve is upward-sloping in the k_1-k_2 plane, as shown in Figure 2.



The case of $1 + \eta > 0$

Figure 2

From Figure 2, the steady state in this model is unique and stable, i.e. starting from any initial k_1 and k_2 the system tends to converge to a unique set of k_1^* and k_2^*. The (strongly) sufficient conditions for this are, along with the general assumptions of this model, (a) the own-elasticity in the asset allocation function g is not smaller than the cross-elasticity[5] and (b) the price elasticity of demand for exports is large enough (though less than unity). The arrows in Figure 2 indicate the possible time paths of k_1 and k_2, and therefore P, the terms of trade.

Although we are mainly concerned with the case of price-inelastic exports, (i.e. $1+\eta>0$) we might briefly report that for the case of $1+\eta \leqq 0$ the analysis is nearly the same. In fact the sufficient conditions for stability and uniqueness of steady state are much weaker: only $e_2 \gtreqq e_1$ is enough. Figures 3 and 4 show the relevant properties of these cases.

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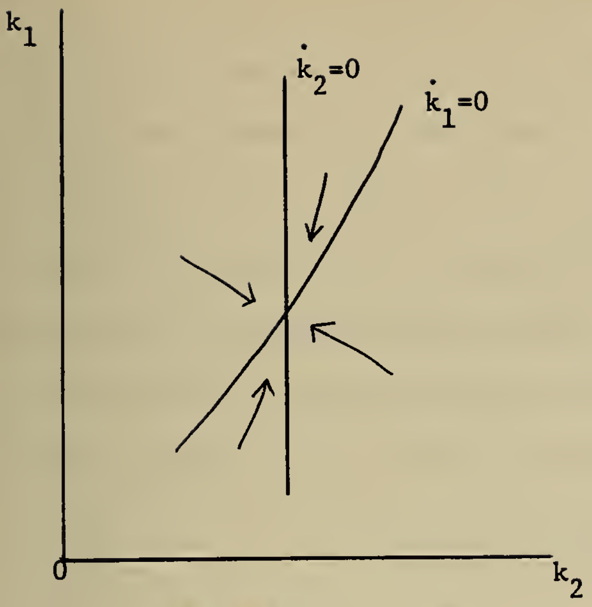
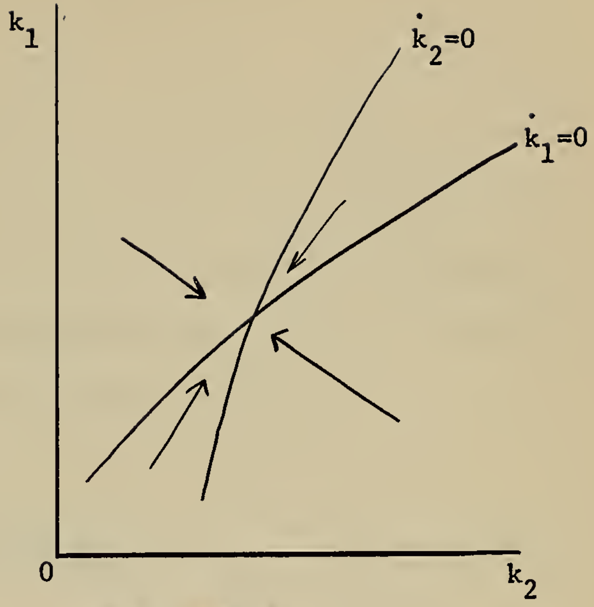
[5] If we use the flow demand function mentioned in f.n. 4, instead of the stock demand function for K_2, the condition is somewhat weaker: $1 + e_2 - e_1 \gtreqq 0$.



The case of $1 + \eta = 0$

Figure 3



The case of $1 + \eta < 0$

Figure 4

Let us now refer to the case when $\mu \neq \lambda$, in particular when the world demand for exports is growing at a sufficiently sluggish rate so that $\mu > \lambda$. The only change involved will be in our equation (28), so that now $X = P^{\eta}e^{\lambda t}$. The analysis of the differential equations of the system becomes now more complicated. But we may briefly report the results around the steady state. In the steady state the rate of growth of capital of each type is constant, so that $\frac{\dot{K}_2}{K_2}\left(= \frac{P^{1+\eta}}{K_2}e^{\lambda t}\right)$ is a constant and $\frac{\dot{K}_1}{K_1}\left(= sk_1^{\alpha-1}k_2^{\beta} - \frac{\dot{K}_2}{PK_1}\right)$ is also a constant. Since from (29) P is still only a function of k_1 and k_2, with the use (30) and (31) we may work out the steady-state rates of growth. They are as follows:

$$\frac{\dot{K}_1}{K_1} = \frac{\mu(1-\alpha-\beta)(1-\eta) + \lambda\beta}{(1-\alpha) - \eta(1-\alpha-\beta)} \tag{42}$$

$$\frac{\dot{K}_2}{K_2} = \frac{\lambda(1-\alpha) - \eta\mu(1-\alpha-\beta)}{(1-\alpha) - \eta(1-\alpha-\beta)} \; . \tag{43}$$

Both of these rates of growth of capital are weighted averages of μ and λ, so that in the steady state k_1 and k_2 decline, remain constant or increase as μ is greater than, equal to or less than λ. The rate of growth of national income is the same as that of K_1. Both of these rates of growth are increasing (decreasing) functions of the absolute value of the price-elasticity of demand for exports if μ is larger (smaller) than λ. An increase in λ, the rate of growth of world demand, however, always increases the rates of growth.

It also turns out that in the steady state

$$\frac{\dot{P}}{P} = \frac{\dot{K}_2}{K_2} - \frac{\dot{K}_1}{K_1} = \frac{(\lambda-\mu)(1-\alpha-\beta)}{(1-\alpha)-\eta(1-\alpha-\beta)}. \tag{44}$$

This means that in the steady state terms of trade are declining (improving) if the world demand is sluggish (bouyant) enough so that μ is larger (smaller) than λ.

We also carried out the local stability analysis for the steady state in this model. The differential equations in the system (involving the rate of growth of the rates of growth of each type of capital) are locally stable if the elasticities in the asset allocation function g are constant and if the own-elasticity, e_2, is not smaller than the cross-elasticity, e_1 Figure 5 shows some of the relevant properties of this system.

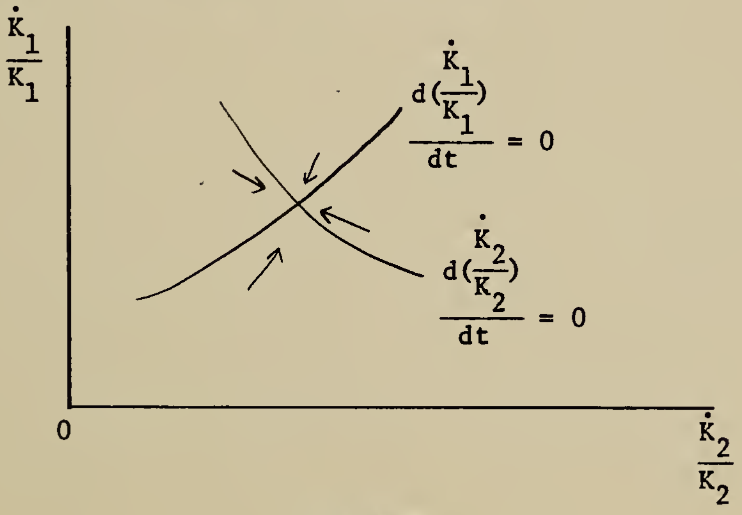

$\frac{\dot{K}_1}{K_1}$
$\frac{d(\frac{\dot{K}_1}{K_1})}{dt} = 0$
$\frac{d(\frac{\dot{K}_2}{K_2})}{dt} = 0$
0
$\frac{\dot{K}_2}{K_2}$

Figure 5

In summary, in this paper we have analyzed in the framework of two simple dynamic models the limitation on growth imposed by the nature of world demand for the country's exports when imports consist of intermediate products or capital goods that are not produced at home. The time paths of output, capital stocks, imports, and terms of trade are analyzed in terms of the properties of the fundamental differential equations of the system.

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